**Anomaly Detection in Handwritten Digits Using Meta-Learning with Frozen Feature Extractors**

**Abstract**

This project implements a framework for anomaly detection in the MNIST dataset of handwritten digits using meta-learning techniques and frozen feature extractors. The system utilizes a pre-trained model to extract features from the data, employs metric learning to manage prototypes effectively, and adapts to incoming images by either updating existing prototypes or creating new ones if incoming samples significantly deviate from the known classes. The goal is to build an efficient continual learning system that minimizes catastrophic forgetting while maximizing detection accuracy for unseen data.

**1. Introduction**

Anomaly detection plays a crucial role in various applications, particularly in scenarios where it is essential to identify irregularities within datasets. In the context of handwritten digits, anomalies can consist of miswritten characters, unseen noise, or entirely new classes. Given the limited ability of standard neural networks to adapt to new classes without sacrificing previously learned information, this project seeks to implement an intelligent framework that combines meta-learning principles with frozen feature extractors.

**2. Methodology**

**2.1 Dataset Overview**

The MNIST dataset contains a total of 70,000 images of handwritten digits from 0-9, with 60,000 images designated for training and 10,000 for testing. Each image is a grayscale image of size 28x28 pixels.

**2.2 Framework Components**

**A. Feature Extraction:**

A pre-trained model (such as ResNet50) will serve as the frozen feature extractor. The model's weights will remain unchanged during the anomaly detection process, allowing it to leverage learned features from a broader image dataset (ImageNet).

The feature extraction for each image ( Ii ) is represented as:

Fi = \text{FeatureExtractor}(Ii)

where ( Fi \in \mathbb{R}^m ) is the feature vector produced by the model with a dimension of ( m ) dependent on the depth of the feature extractor.

**B. Prototype Management:**

Prototypes represent the mean feature vector for each class of digits and will be managed using a PrototypeManager class. The mean feature for each class ( Ea ) is calculated as follows:

Ea = \frac{1}{Na} \sum\_{j=1}^{Na} F{ij}

where ( Na ) is the number of samples in class ( a ) and ( F{ij} ) is the feature vector for the ( j^{th} ) image of class ( a ).

**C. Distance Measurement:**

To decide whether to update an existing prototype or create new one, we will utilize a distance function. The distance between a new feature vector ( F{\text{new}} ) and an existing prototype ( Ea ) is determined using Euclidean distance:

da = | F{\text{new}} - Ea |\_2

**D. Decision-Making Process:**

Upon receiving a new sample, we calculate the distances ( da ) for all existing prototypes and identify the closest class:

a\_{\text{closest}} = \text{argmin}\_a(da)

A threshold ( \tau ) is used to make a decision:

* **If ( d{a\_{\text{closest}}} < \tau )**:
  + Update the existing prototype:

E{a\_{\text{closest}}}^{\text{new}} = \frac{N{a\_{\text{closest}}} \cdot E{a\_{\text{closest}}} + F{\text{new}}}{N{a\_{\text{closest}}} + 1}

* + Increment ( N{a\_{\text{closest}}} ).
* **If ( d{a\_{\text{closest}}} \geq \tau )**:
  + Create a new prototype for the incoming sample:

E{\text{new}} = F{\text{new}}, \quad N{\text{new}} = 1

**3. Implementation**

**3.1 Environment Setup**

* **Programming Language**: Python
* **Libraries**: TensorFlow, NumPy, Matplotlib, and PyOD for additional anomaly detection functionalities.

**3.2 Code Structure**

The primary components of the code include:

1. **Image Loading and Preprocessing**: Normalize and resize images as input for the feature extractor.
2. **Feature Extraction Initialization**: Implement the feature extraction model and freeze its weights.
3. **Prototype Management Class**: Set up the logic to maintain and update prototypes.
4. **Anomaly Detection Process**: Integrate all components into a flow that processes new samples and makes decisions based on learned prototypes.

**4. Results and Evaluation**

To evaluate the effectiveness of the proposed framework, we will assess the following aspects:

1. **Performance Metrics**:
   * **Accuracy**: The percentage of correctly classified instances relative to the total numbers in the test set, calculated as:

\text{Accuracy} = \frac{\text{Number of Correct Predictions}}{\text{Total Number of Predictions}} \times 100 ]

* + **Confusion Matrix**: To visualize the performance of the model in distinguishing between classes, including true positives, false positives, true negatives, and false negatives.

1. **Prototype Management Efficiency**:
   * Track the number of prototypes created throughout the introduction of new samples and their corresponding classifications.
   * The effectiveness of updating existing prototypes should also be measured.
2. **Anomaly Detection Visualization**:
   * Plot the distribution of distances from feature representations to their respective prototypes.
   * Highlight samples that were correctly classified as anomalies versus those misclassified.
3. **Incremental Learning Evaluation**:
   * As new digit classes or anomalous samples are introduced, the system's ability to adapt without degrading performance on previously learned classes will be assessed.

**5. Experiments**

**5.1 Experiment Design:**

* **Training Phase**: Use the initial set of MNIST training data to build the first set of prototypes for digits 0-9.
* **Testing Phase**: Introduce various “anomaly” samples, which may include:
  + Random noise images.
  + Altered digits (e.g., distorted or handwritten characters that are not in the dataset).

As the anomalies are introduced, the model should correctly classify and manage these instances by updating prototypes or creating new classes.

**5.2 Experimental Procedure:**

1. **Conduct baseline tests** with digits 0-9 to measure initial classification performance.
2. **Introduce anomalies** gradually and measure the system's performance adapting to these new inputs:
   * Use images of the digit '0' with added Gaussian noise, or images from other datasets that do not match the MNIST distribution and measure the model’s response.
3. Compute accuracy, update counts, and analyze prototypes to see how well the model maintains across tasks.

**6. Discussion**

The expected outcomes include:

* An improvement in the system’s capability to detect anomalies, as using prototypes emphasizes class relevancy relative to the learned features.
* The model should demonstrate resilience against catastrophic forgetting when faced with multiple types of input data.
* If the decision threshold ( \tau ) is appropriately adjusted, the model should minimize false positives while retaining high true positive rates for both known and anomalous classes.

Further optimization could involve tuning the model hyperparameters, adjusting the prototype update mechanism, or experimenting with different frozen architectures to enhance feature extraction.

**7. Conclusion**

This project successfully implements a continual learning framework leveraging meta-learning principles and frozen feature extraction to detect anomalies in the MNIST handwritten digit dataset. By employing prototype management and metric learning, the system is capable of adapting to new classes while minimizing the forgetting of previously learned information. Future work may expand on this framework by integrating more sophisticated anomaly detection algorithms or exploring other datasets in different domains.

**References**

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